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Problems of Science. By FEDERIGO ENRIQUES. Authorized translation by KATHARINE ROYCE with an introductory note by JOSIAH ROYCE. xvi+392 pages. The Open Court Publishing Co., Chicago, 1914. $2.50.

The first edition of the Italian text of the *Problema della Scienza* of Professor Enriques appeared in 1906. It has already become known to a wide circle of European students. It is a pleasure to welcome its appearance in English.

The book contains six chapters treating in order the following topics: the general problem of knowledge and related matters; facts and theories and their interactions; the general problems of logic; the philosophical and psychological questions which are naturally raised in connection with the science of geometry; mechanics, its objective significance and the psychological development of its principles; the extension of mechanics into physics and the relation of the mechanical hypothesis to the phenomena of life.

This book is of quite unusual value. It is written by a mathematician and consequently takes proper account of recent mathematical developments. It contains a masterly analysis of the problems of science, especially in their relation to matters of mathematical expression and of philosophical import. It merits the close attention alike of physicists and philosophers and mathematicians.

R. D. CARMICHAEL.

Geometry of Four Dimensions. By HENRY PARKER MANNING, Ph.D., Associate Professor of Pure Mathematics in Brown University. The Macmillan Company, New York, 1914. ix+348 pages. $2.00.

The production of this work, which has taken several years, has evidently been a labor of love. The result is a book physically handsome, beautiful in its content, creditable alike to the publishers and to the author. The introduction of 22 pages is itself worth more than the price of the volume. We have here, besides a clear indication of the aim and point of view of the present work along with reasons for its methods and procedure, a fair evaluation of hyperspace studies in general, and an admirable sketch of the origin and earlier developments of the subject with a clew to its literature to date. It is a curious fact that, although space dimensionality was a subject of thought with philosophers and mathematicians from Aristotle down, yet geometry of hyperspace is still under a hundred years old, the earliest contribution to the subject being, Professor Manning tells us, that of Möbius in his *Calcul*, 1827, and even Möbius thought that 4-dimensional space could not be "*gedacht.*" Why was the beginning so tardy? "The general notion," says Professor Manning, "that geometry is concerned only with objective external space made the existence of any kind of geometry seem to depend upon the existence of the same kind of space." The answer is good so far as it goes but it might have gone farther and deeper. It might well have been made clear that there neither is nor can be a mathematical geometry of sensible space; that the subject of mathematical geometry is indeed objective external space but is a conceptual space of the kind and not a sensible one; and that the space of 4-dimensional or n-dimensional geometry has the same kind of existence as has the space of ordinary solid geometry. What prevented the rise of multi-dimensional geometry was, not bad logic, but bad psychology; and it is bad psychology that still leads many mathematicians to apologize for using the language of such geometry because it seems to imply belief in the existence of a corresponding space. The philosophy of mathematics has had many devotees. Of the logic of mathematics there is a large and growing literature. The psychology of mathematics has been neglected. Here is a great opportunity. Who will improve it?

"Our plane and solid geometries," says Professor Manning, "are but the beginnings of this science" (of geometry). He is right. Just as a student learns the real meaning of analysis only when he studies the calculus, so the real nature of geometry shows itself only in the light of the theory of hyperspace and especially that of four dimensions. The book in hand provides a natural transition. The methods are those of the ordinary geometry of the secondary school or of college, and no mathematical knowledge is presupposed beyond that of the usual elements of solid geometry. Algebra is not employed. The method is that of so-called pure or synthetic geometry. Beyond the explicit introduction of two axioms concerned with the relation of collinearity, no special stress is given to the matter of postulates or of rigor for rigor's sake. Instead, the student is presented with a four-dimensional structure built upon familiar foundations by familiar methods. In this respect and in its limitation to space of four dimensions, the work compares both favorably, as being simpler, and unfavorably, as being less rich, with the *Mehrdimensionale Geometrie* of Schoute, where hyperspaces of every dimensionality are studied by several elementary methods both analytic and synthetic. One who had read Schoute's book would not need to read that of Manning. On the other hand many a student not qualified to read the former can read the latter with pleasure and profit. The work is a point geometry in the sense that the point is the undefined element, all other figures being regarded as classes of points. The procedure being that of metric geometry, one misses the beautiful interplay of such dual developments as arise from the projective point of view out of the reciprocity of point and hyperlane, and of line and plane. The first 220 pages are pangeometric in the Lobachevskian sense, no question of parallels intervening in course of the first five chapters. Otherwise the theory is Euclidean except for some sections devoted to such themes as the geometry of the hypersphere and that of the hyperplane at infinity. It is notable, by the way, that the phrase "at infinity" and equivalent phrases are used merely to facilitate talk about parallelism and do not imply the existence of infinite numbers or distances, no infinite region or element being posited.

In the first chapter, concerned with foundations, certain familiar concepts and theorems are presented with extraordinary care and clarity to fortify the reader against the approaching shock of such 4-space relations and possibilities as transcend experience. Perhaps the author is a bit unfortunate in speaking of such relations—the fact, *e. g.*, that in general two planes have one and but one common point—as "contradicting" experience. To say that it is transcendence rather than contradiction seems far enough from verbal quibbling. It seems a pity that the notion of a line pencil was not available so that the very beautiful theorem on page 61 could have been made to read: *Two independent planes contain, each of them, one and but one pencil of lines such that any two of the lines are coplanar.* Denote the planes by α and β. Then we may say with Professor Manning that "α and β are covered with these lines." But it is somewhat ambiguous to say that the planes "might be said to consist of them." They consist of the points of the lines but not of the lines as lines.

It is in the second chapter, devoted to perpendicularity, that the beginner will realize that the narrow shell of his familiar geometric conceptions is bursting asunder and letting in the light of vistas of which he had never dreamed. And the surprise and joy will attend him throughout the book, being indeed renewed and deepened when he enters upon the doctrine of parallelism in chapter VI.

The normal development is interrupted in chapter III in order to point out the possibility and nature of what is called "Point Geometry," that is, the theory of the angles at a 4-space point. This is a 3-dimensional geometry, not of, but *within*, 4-space. The name is hardly felicitous in view of the fact that the entity taken for element is not the point nor indeed the angle but the line or half-line. Possibly a better name would have been line geometry or angle geometry of a 4-space point. It might have been indicated, too, that there is possible a kind of reciprocal 3-dimensional geometry of the hyperplanes enveloping a point, as well as, matching the line geometry of a hyperplane, a 4-dimensional geometry of the planes enveloping a point of 4-space. Analogous suggestions occur in noticing the place here accorded to what the author calls "Edge Geometry," that is, the 2-dimensional theory of planes or half-planes having a line or edge in common.

Chapters IV, V, VI, VII, and VIII deal respectively with symmetry, order, and motion; hyperpyramids, hypercones, and the hypersphere; Euclidean geometry, figures with parallel elements; measurement of volumes and hypervolumes in hyperspace; and the regular polyhedroids.

The book ought to be in the library of every teacher of high-school mathematics and would serve admirably for use in an advanced undergraduate elective course in pure geometry.

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